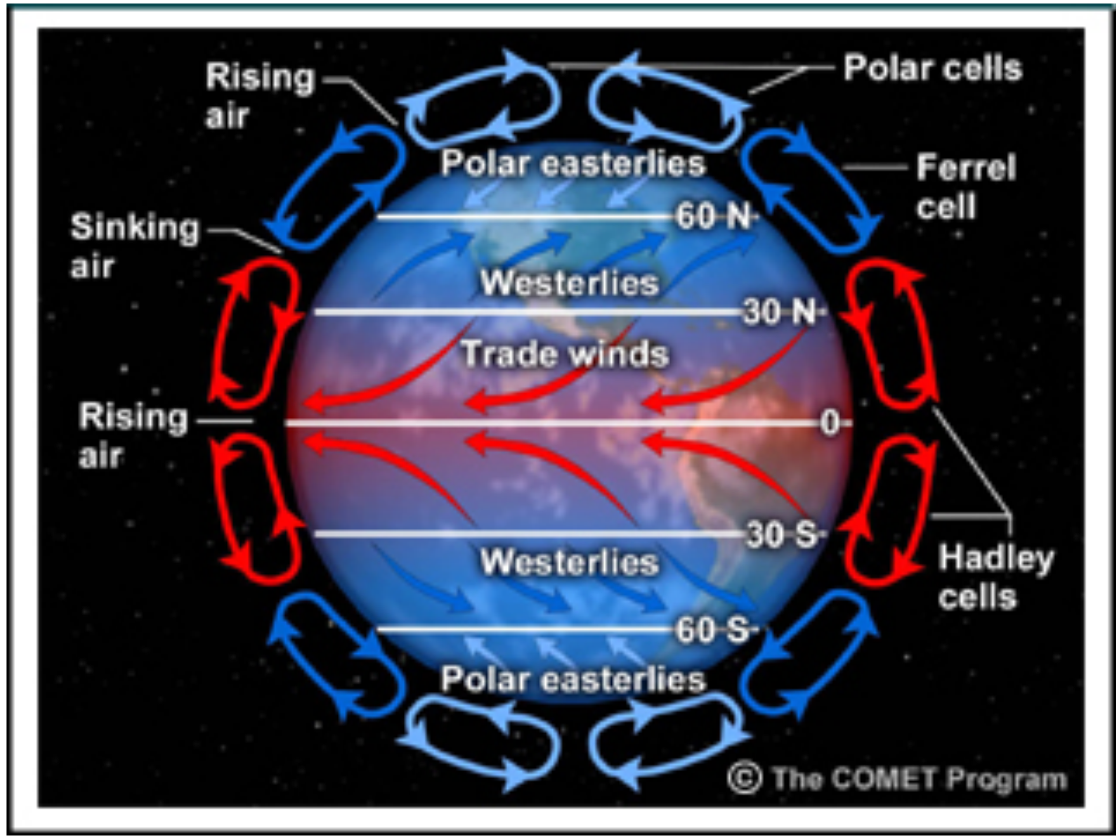
- Description of the zonal mean tropical overturning (or Hadley) circulation
- Some simple dynamical and thermodynamic models of the Hadley circulation*
- The Hadley circulation in a global circulation context

Goal: Understand the dynamics and thermodynamics of the Hadley circulation

*Held, Isaac M., Arthur Y. Hou, 1980: Nonlinear Axially Symmetric Circulations in a Nearly Inviscid Atmosphere. *J. Atmos. Sci.*, **37**, 515–533.

Overview of the Hadley circulation

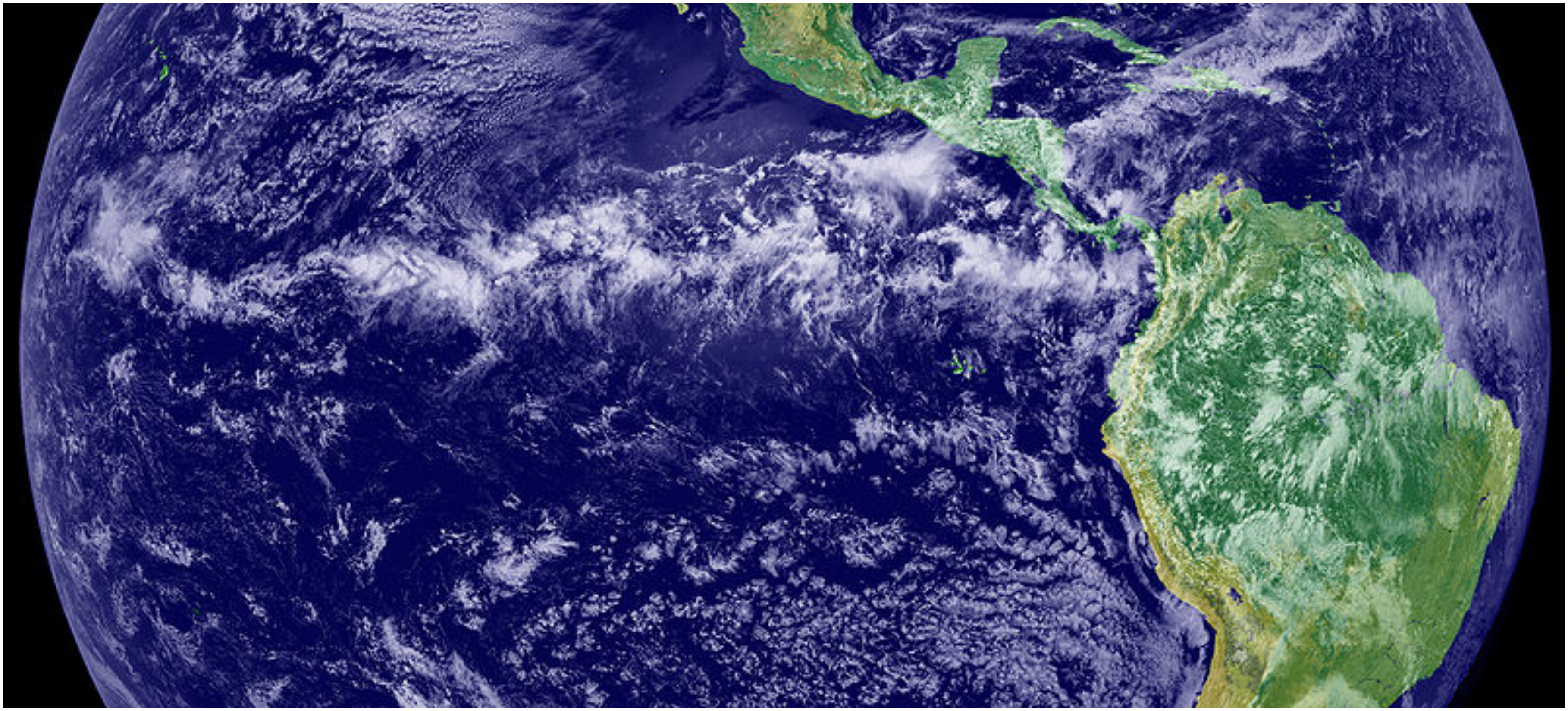
- The annual-mean Hadley circulation comprises two cells, characterized by rising motion near the equator, poleward flow at roughly 10-15 km, descending motion over the subtropics, and equatorward flow near the surface
- The Hadley circulation is strongly tied to the trade winds, tropical convergence zones, subtropical deserts, and jet streams



Brief history

- Edmond Halley (1656-1742): theorized that near-equatorial solar heating generates upward motion that is replaced by inflowing air masses from adjacent latitudes and that the easterly component of the trades arises from differential heating of the atmosphere over the course of a day
- George Hadley (1685-1768): Earth’s rotation plays a role in the direction of air masses moving relative to the earth’s surface
- William Ferrel (1817-1891): angular momentum, rather than linear momentum, is conserved in the absence of frictional dissipation

Very wet:
Intertropical Convergence Zone (ITCZ)



East Pacific ITCZ from the Geostationary Operational Environmental Satellite 11 (GOES-11)
July 2000

And very dry:
Sahara Desert

Trade winds

Herman Moll, 1736

The Horse Latitudes

- The subsiding subtropical edges of the Hadley circulation are regions of high surface pressure and light winds called the "horse latitudes"
- *Why are they called "horse latitudes"?*
  - Drifting ships in the calm wind region would toss horses overboard to conserve resources. Unlikely.
  - "Dead horse" ritual: a sailor's receipt of a pay advance from shipmaster resulted in a period of debt ("dead horse" time), at the end of which a straw-stuffed horse effigy was thrown overboard—for a westbound ship from Europe, this usually occurred upon reaching the subtropics

Source unknown

The Doldrums

- Near-equatorial zone of calm winds
- Rime of the Ancient Mariner (1797-98) by Samuel Taylor Coleridge:

All in a hot and copper sky,
The bloody Sun, at noon,
Right up above the mast did stand,
No bigger than the Moon.
Day after day, day after day,
We stuck, nor breath nor motion;
As idle as a painted ship
Upon a painted ocean.

"Canoes in the Doldrums" by Herb Kawainui Kane

Mass streamfunction

Consider the mass conservation (continuity) equation in ϕ, λ, and p:

$$\nabla \cdot \mathbf{v} = \frac{1}{R_e \cos\lambda}\frac{\partial u}{\partial \phi} + \frac{1}{R_e \cos\lambda}\frac{\partial (v\cos\lambda)}{\partial \lambda} + \frac{\partial \omega}{\partial p} = 0$$

Integrating this equation over all longitudes gives:

$$\int_0^{2\pi} \nabla \cdot \mathbf{v}\, d\phi = \frac{u(2\pi) - u(0)}{R_e \cos\lambda} + \int_0^{2\pi} \left(\frac{1}{R_e \cos\lambda}\frac{\partial (v\cos\lambda)}{\partial \lambda} + \frac{\partial \omega}{\partial p} \right) d\phi = 0$$

$$\frac{1}{R_e \cos\lambda}\frac{\partial (\langle v \rangle \cos\lambda)}{\partial \lambda} + \frac{\partial \langle \omega \rangle}{\partial p} = 0 \qquad \langle \ldots \rangle = (2\pi)^{-1}\int_0^{2\pi} \ldots d\phi$$

The 2D zonal-averaged flow in the latitude-pressure plane can be expressed in terms of a *mass (Stokes) streamfunction*, ψ, defined such that:

$$\langle v \rangle = \frac{g}{2\pi R_e \cos\lambda}\frac{\partial \psi}{\partial p} \qquad \langle \omega \rangle = -\frac{g}{2\pi R_e \cos\lambda}\frac{\partial \psi}{R_e \partial \lambda}$$

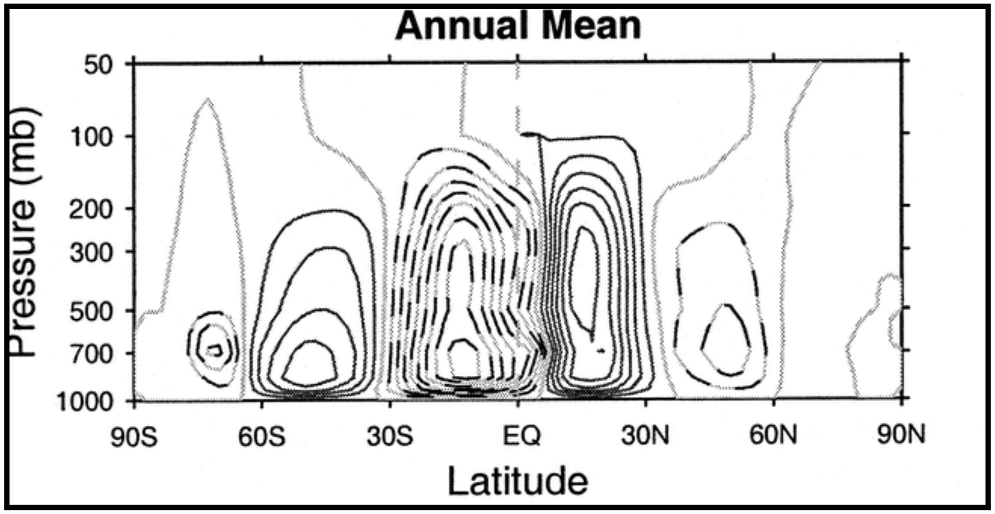
Starting the integral at the top of the atmosphere where $\psi \equiv 0$

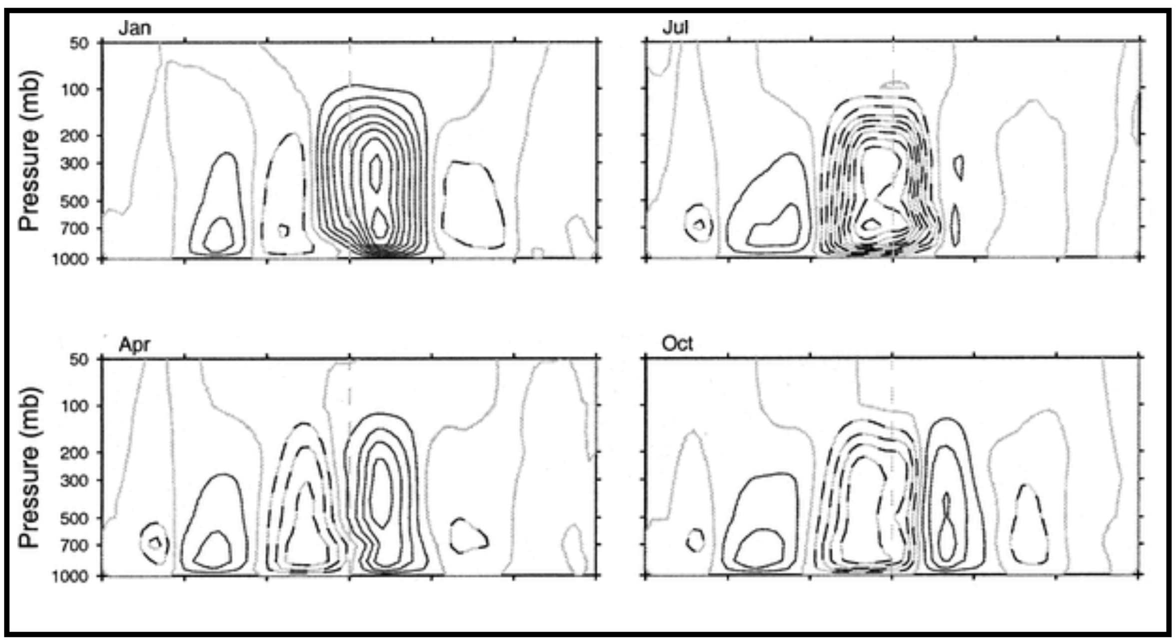
Thus:

$$\psi(\lambda, p) = \psi(\lambda, p_0) + (2\pi R_e \cos\lambda) g^{-1} \int_{p_0}^{p} \langle v \rangle dp' \Rightarrow \psi(\lambda, p) = (2\pi R_e \cos\lambda) g^{-1} \int_{p_{toa}}^{p} \langle v \rangle dp'$$

From dimensional considerations, ψ is seen to have dimensions of mass/time.

Streamfunction climatology





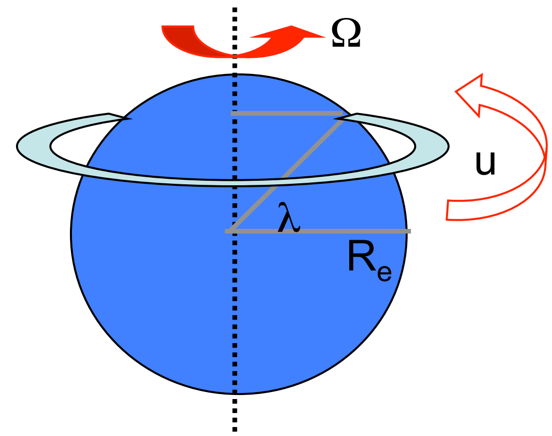
Dima and Wallace, 2003

- The annual mean streamfunction is slightly asymmetric w.r.t. the equator
- Seasonally, the streamfunction experiences a dramatic reversal
  - *For solsticial seasons:* a single dominant Hadley cell, circulating clockwise (solid contours) in January and counterclockwise in July
  - *For equinoctal seasons:* two Hadley cells of comparable (but weaker) intensity

An angular momentum argument for the mean zonal circulation

The absolute angular momentum per unit mass of atmospheric flow flow, $\vec{M}$, is given by:

$$\vec{M} = \vec{r} \times \vec{v}_{inertial} = \vec{r} \times (\vec{v} + \vec{\Omega} \times \vec{r})$$



Let's assume a ring of air at a latitude λ with a zonal windspeed *u(λ)*. Then the magnitude of $\vec{M}$ is:

$$M(\lambda) = R_e \cos\lambda [u(\lambda) + \Omega R_e \cos\lambda]$$

Suppose this ring of mass originated at the equator with zero windspeed. If angular momentum per unit mass is conserved, then:

$$M(0) = M(\lambda) \Rightarrow \Omega R_e^2 = \Omega R_e^2 \cos^2\lambda + u(\lambda) R_e \cos\lambda$$

$$\Rightarrow u(\lambda) = \frac{\Omega R_e \sin^2\lambda}{\cos\lambda}$$

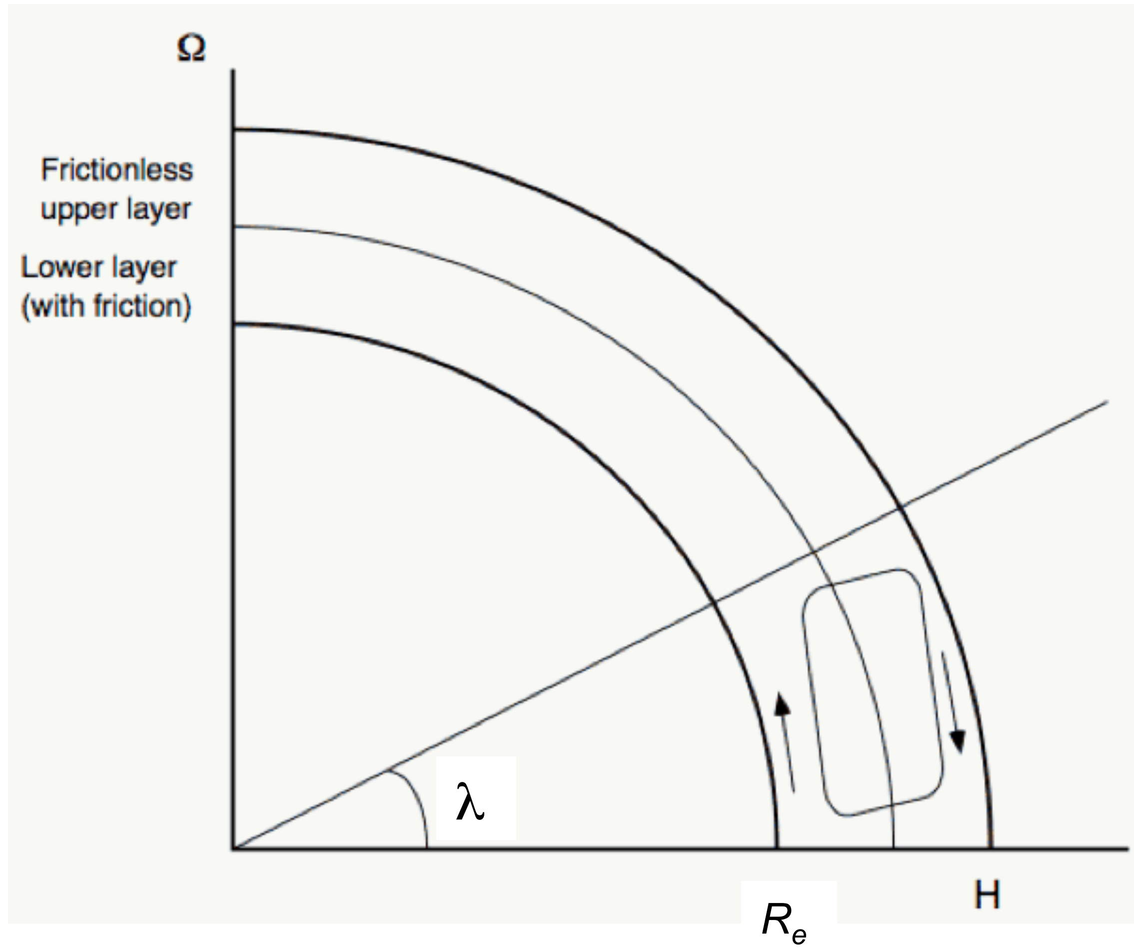
At 20° from the equator, this implies a windspeed of 56 ms^{-1}, while at 30°, the windspeed is 127 ms^{-1}. The observed subtropical jet windspeed is roughly 35-40 ms^{-1}: thus, while the zonal windspeed implied by angular momentum conservation at 20° is reasonable, the value at 30° is not.

The Held-Hou Model (1980)

The Held-Hou model of the Hadley circulation considers the set-up as shown.

Key Assumptions:

- Two layer zonally-symmetric flow
- The zonal flow is steady and “balanced”
- The upper poleward branch conserves angular momentum (no frictional dissipation)
- Surface winds are small (friction in the lower layer)
- Negligible thermal diffusion



The Held-Hou Model (1980)

Let's consider the hydrostatic and meridional momentum equations in pressure coordinates under the assumptions of the Held-Hou model:

$$\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho}$$

$$\frac{1}{R_e}\frac{\partial \Phi}{\partial \lambda} + 2\Omega u \sin\lambda = 0$$

We consider here a geostrophic or thermal wind balance. Differentiating the momentum equation with respect to pressure gives:

$$\frac{\partial}{\partial p}\frac{\partial \Phi}{\partial \lambda} = -2\Omega R_e \sin\lambda \frac{\partial u}{\partial p}$$

On the other hand, differentiating the hydrostatic equation with respect to latitude—and noting that, in doing so, *p* is fixed—gives:

$$\frac{\partial}{\partial \lambda}\frac{\partial \Phi}{\partial p} = -\frac{\partial}{\partial \lambda}\left(\frac{1}{\rho}\right)_p = -\frac{\partial}{\partial \lambda}\left(\frac{RT}{p}\right)_p = -\frac{R}{p}\left(\frac{\partial T}{\partial y}\right)_p = -\frac{R}{p}\frac{T}{\theta}\left(\frac{\partial \theta}{\partial y}\right)_p$$

Here the definition of potential temperature is used, again noting *p* is constant.

Equating the mixed partial derivatives of geopotential yields:

$$-2\Omega R_e \sin\lambda \frac{\partial u}{\partial p} = -\frac{R}{p}\frac{T}{\theta}\left(\frac{\partial \theta}{\partial y}\right)_p = -\frac{1}{\rho}\frac{\partial \ln\theta}{\partial \lambda} \;\longrightarrow\; 2\Omega \sin\lambda \frac{\partial u}{\partial z} = -g\frac{\partial \ln\theta}{\partial y}$$

Convert to *z* using hydrostatic balance and let $dy = R_e d\lambda$.

The Held-Hou Model (1980)

Using the expression for *u* derived from angular momentum considerations, and making small angle approximations for sine and cosine, gives:

$$u(\lambda) = \frac{\Omega R_e \sin^2 \lambda}{\cos \lambda} \approx \frac{\Omega R_e (\lambda^2)}{1} = \frac{\Omega (R_e \lambda)^2}{R_e} \Rightarrow u(y) \approx \frac{\Omega y^2}{R_e}$$

Since the lower layer windspeed is ~0, the vertical shear of the zonal wind can be approximated as:

$$\frac{\partial u}{\partial z} \approx \frac{\Omega y^2}{R_e H}$$

Thus:

$$\frac{\partial \ln \theta}{\partial y} = -\frac{2\Omega \sin \lambda}{g} \frac{\partial u}{\partial z} \Rightarrow \frac{1}{\theta_0} \frac{\partial \theta}{\partial y} \approx -\frac{2\Omega y}{R_e g} \left(\frac{\Omega y^2}{R_e H} \right)$$

and:

$$\int_{\theta_{M0}}^{\theta_M} d\theta = -\frac{2\Omega^2 \theta_0}{R_e{}^2 gH} \int_0^y y'^3 dy' \quad \longrightarrow \quad \theta_M = \theta_{M_0} - \frac{\Omega^2 \theta_0}{2R_e{}^2 gH} y^4$$

The subscripts *M* and M_0 denote that this solution is consistent with angular momentum balance.

The Held-Hou Model (1980)

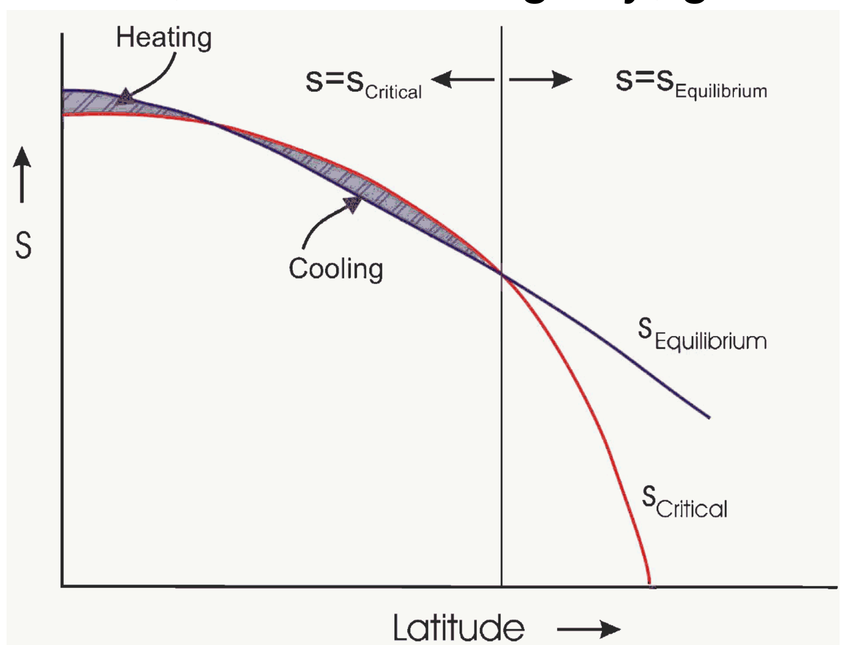
We can also estimate the potential temperature profile as predicted by radiative constraints. To leading order, this profile has a form approximated by:

$$\theta_E(\lambda) = \theta_{EG} - \frac{2}{3}\Delta\theta P_2(\sin\lambda)$$

Here, θ_{EG} is an upper tropospheric global mean radiative equilibrium potential temperature, $\Delta\theta$ is the pole-to-equator radiative equilibrium potential temperature difference, and P_2 is the "2nd Legendre polynomial", defined such that $P_2(x)=1/2(3x^2 - 1)$.

Considering again the small angle approximation, and converting to y, gives:

$$\theta_E \approx \theta_{E_0} - \frac{\Delta\theta y^2}{R_e^{\ 2}}$$



The profiles on the right illustrate θ_M (red curve) and θ_E (blue curve). If the net atmospheric heating is assumed to be the difference θ_E - θ_M, then close to the equator, net warming is taking place, while at somewhat higher latitudes, net cooling occurs.

The Held-Hou Model (1980)

If we assume that over the width of the Hadley cell the net warming cancels the net cooling, then

We're formally calculating:

$$\int_0^Y \theta_E(y) - \theta_M(y)dy = 0 \qquad \int_0^Y \frac{d\theta}{dt}dy = 0\,;\ \frac{d\theta}{dt} = \frac{\theta_E - \theta}{\tau_E}\,;\text{where } \theta = \theta_M$$

Performing the above integration gives:

$$(\theta_{E_0} - \theta_{M_0}) + \frac{\Omega^2\theta_0}{10R_e^{\ 2}gH}Y^4 - \frac{\Delta\theta}{3R_e^{\ 2}}Y^2 = 0$$

One root, Y = 0, is disregarded.

We also note that $\theta_E(Y) = \theta_M(Y) \Rightarrow (\theta_{E_0} - \theta_{M_0}) + \dfrac{\Omega^2\theta_0}{2R_e^{\ 2}gH}Y^4 - \dfrac{\Delta\theta}{R_e^{\ 2}}Y^2 = 0$

Thus,

$$Y = \left(\frac{5\Delta\theta gH}{3\Omega^2\theta_0}\right)^{1/2} \quad \text{and} \quad \theta_{E_0} - \theta_{M_0} = \frac{5\Delta\theta^2 gH}{18R_e^{\ 2}\Omega^2\theta_0}$$

For earth's upper atmosphere, with θ_0 = 255K, $\Delta\theta$ = 40K, and H = 12 km, so Y = 2400 km (or roughly 22° latitude) and $\theta_{E0} - \theta_{M0}$ = 0.9K.

The Held-Hou Model (1980)

For y > Y, the difference θ_E - θ_M > 0, implying net warming of the atmosphere. However, this is not physically realistic. So, for this region,

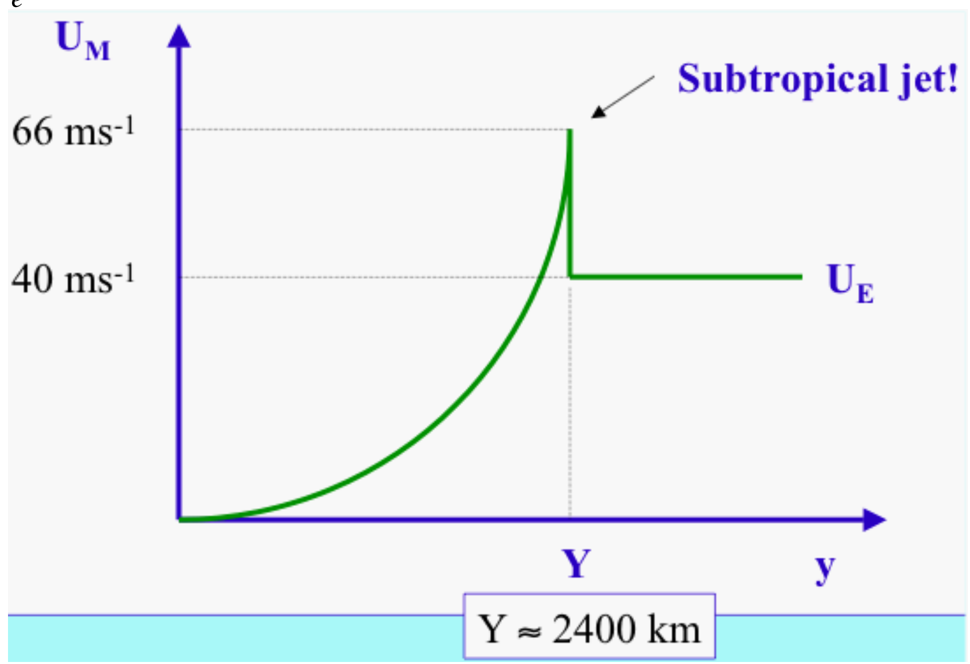
$$\theta = \theta_E \approx \theta_{E_0} - \frac{\Delta\theta y^2}{R_e^{\,2}}$$

At 45°, the small angle approximation is only off by 10%.

Applying thermal wind balance here gives:

$$\frac{\partial \ln\theta_E}{\partial y} = -\frac{2\Omega\sin\lambda}{g}\frac{\partial u}{\partial z} \Rightarrow -\frac{2\Delta\theta}{\theta_0}\frac{y}{R_e^{\,2}} \approx -\frac{2\Omega y}{gR_e}\frac{u}{H}$$

$$u(y) = \frac{gH\Delta\theta}{R_e\Omega\theta_0}$$



The Held-Hou Model (1980)

We finally consider the vertical and meridional winds.

$$w_E \frac{\partial \theta}{\partial z} \approx \frac{\theta_{E_0} - \theta_{M_0}}{\tau_E} \Rightarrow w_E \approx \left(\frac{\partial \theta}{\partial z}\right)^{-1} \frac{\theta_{E_0} - \theta_{M_0}}{\tau_E} = \frac{g}{\theta_0} N^{-2} \frac{\theta_{E_0} - \theta_{M_0}}{\tau_E}$$

Recall this balance in the thermodynamic equation from the tropical scaling analysis.

If the radiative relaxation time is 15 days and Brunt-Väisälä frequency *N* is 0.01 s^{-1}, $w_E \approx 0.00027$ m s^{-1}. If we assume that the vertical velocity vanishes at the z = 0 and z = H of the atmosphere and w_E is the value at w_E at H/2, then:

$$w(z) \approx 4 w_E \frac{z}{H}\left(1 - \frac{z}{H}\right)$$

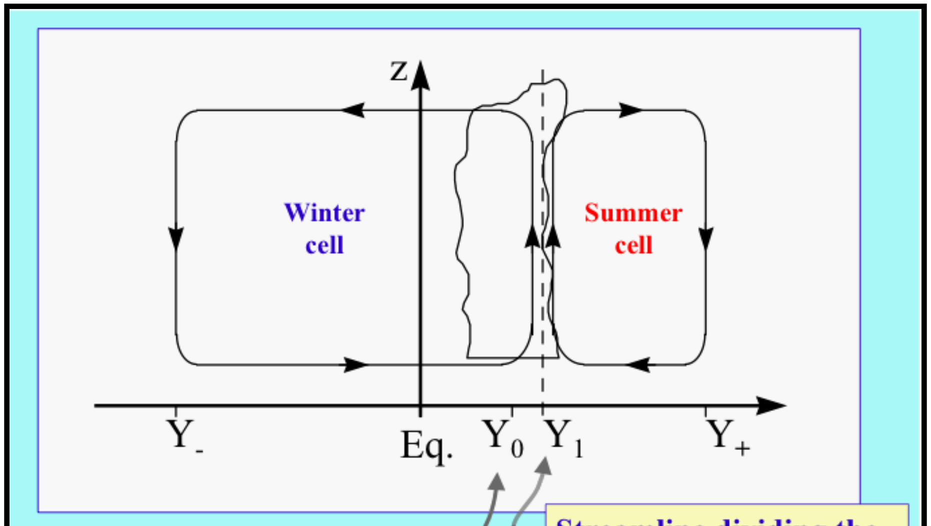
By continuity of the flow:

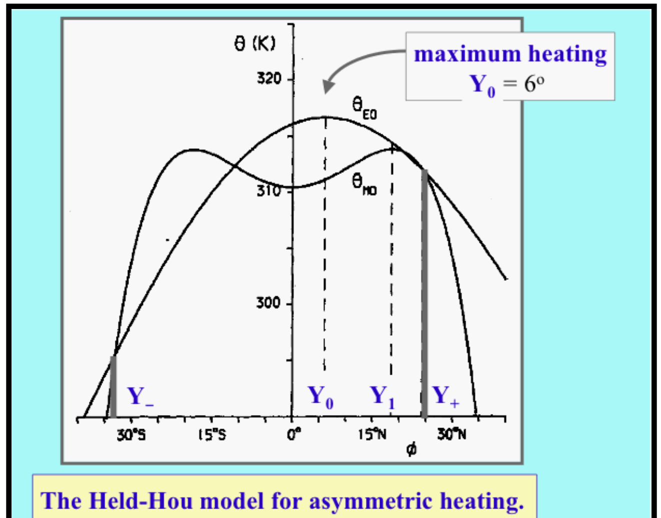
$$\left.\frac{\partial w}{\partial z}\right|_y + \left.\frac{\partial v}{\partial y}\right|_z \approx 0 \Rightarrow v(y,z) \approx -\left[\frac{4 w_E}{H}\left(1 - \frac{2z}{H}\right)\right] Y \; ; y \leq \frac{Y}{2}$$

At Y = Y/2 and z = H, v $\approx 2w_E Y/H \approx 0.01$ ms^{-1}. In the real atmosphere, *v* is of order 1 ms^{-1}, so the modeled Hadley circulation is much too weak.

Extensions to the Held-Hou Model

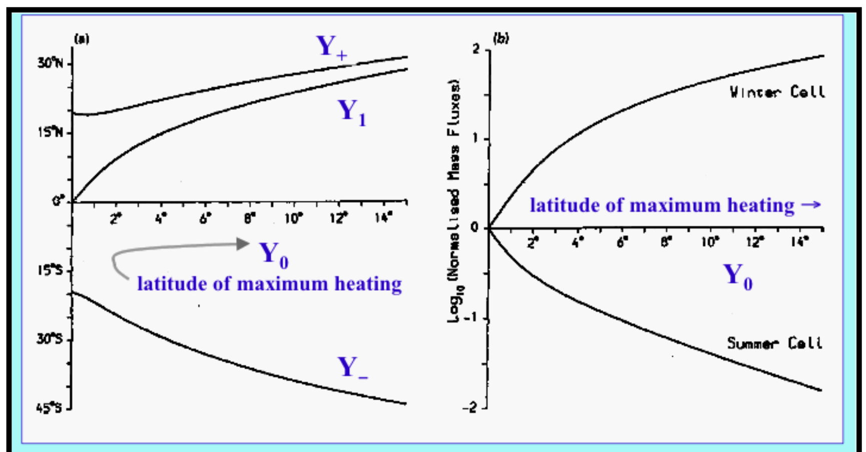
- e.g., Lindzen and Hou (1988): asymmetric heating about the equator





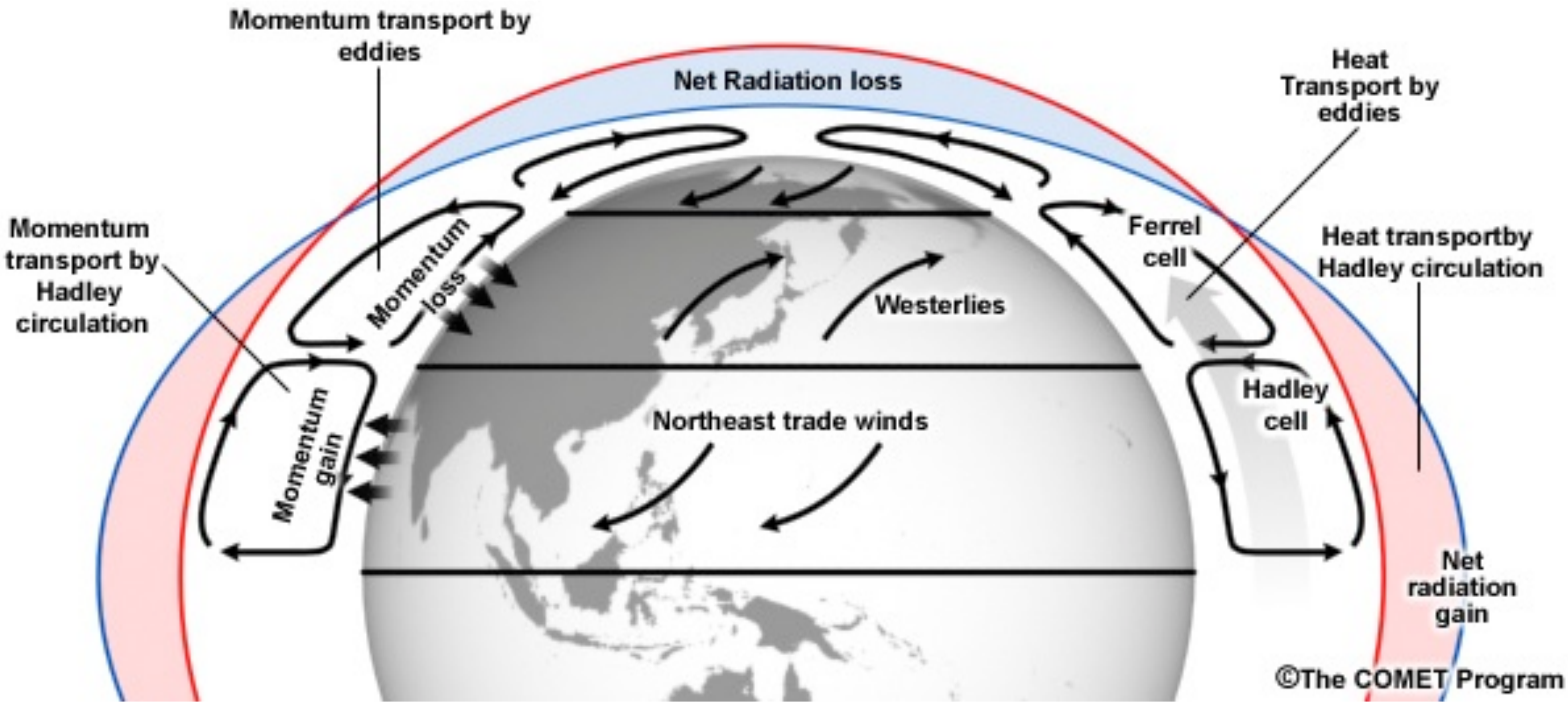
The Held-Hou model for asymmetric heating.

(From James, 1994)



(a) Poleward extent of the summer and winter circulations and of the latitude of the dividing streamline. (b) Mass flux carried by the winter and summer cells.

Role of the Hadley circulation in the General Circulation



- Recall that net TOA heating occurs in the tropics; thus, the tropics is a driver of the general circulation
- The Hadley circulation is responsible for both heat and momentum transport in the tropical atmosphere [note that the mean easterlies near the surface result in momentum gain by the atmosphere in the tropics]